**Trajectory planning of a two-link rehabilitation robot arm**

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**Abstract**— In this paper, the trajectory planning of a two-link rehabilitation robot arm is addressed. Firstly, a Cartesian planning of the trajectory is defined to perform a proper motion exercise for the patient’s arm. Secondly, a trajectory planning in joint space is studied to obtain a similar Cartesian path with less computational complexity. The robot arm analyzed is the two degrees of freedom serial structure of the wire-based robot MariBot, which was designed and realized at the Robotics Laboratory of DIMEG (Department of Innovation in Mechanics and Management, University of Padua, Padua, Italy). 

Keywords: trajectory planning, joint space, Cartesian space, path control, robot arms, wire-driven rehabilitation robot.

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**I. Introduction**

The rehabilitation of post-stroke hemiplegic subjects aims to reduce motor impairment and to improve functional use of the affected limbs to allow early reintegration into social and domestic life. Robot-aided therapy increases the efficiency of the therapist by reducing the labor-intensive aspect of physical rehabilitation [1], [2], [3] and can strongly affect the recovery process by generating repetitive movements in passive and active mode [4].

In passive mode the robot moves along a given trajectory while the patient relaxes, whereas in active mode the patient actively participates to the exercise. Several robotic prototypes designed for upper limb rehabilitation can be found in the literature. One major example is the MIT-MANUS [5], [6]. Its first version was characterized by a two degrees of freedom (d.o.f.) serial structure to rehabilitate the shoulder and the elbow of the patient, then a three d.o.f. system was added for the wrist rehabilitation. The first clinical tests [7] of the 5 d.o.f. complete structure demonstrated a satisfactory contribution to the reduction of the motor impairment of the patient. Another example of rehabilitation robot is the MIME [8] characterized by a 6 d.o.f. Puma robot arm placed on the patient’s paretic side for his upper-limb rehabilitation.

At the Robotics Laboratory of DIMEG (Department of Innovation in Mechanics and Management, University of Padua, Italy) our Research Group designed and realized two wire-based robots for upper limb rehabilitation: the NeReBot (NEuroREhabilitation roBOT, [9]) and its evolution, the MariBot (MARIsa roBOT, [10], [11]). These robots were conceived to deliver the therapy starting from a very early stage, right after the stroke event. The choice of employing the wire-drive philosophy allowed to gain a good compromise between machine simplicity and range of motion, realizing an affordable clinical solution. As a matter of fact, the use of wires avoids the users to be hurt in case of accidental collision, allows implementing a light and simple parallel structure robot and gives the patient a pleasant feeling of being guided and not restrained by the machine.

Industrial manipulator tasks are characterized by repetitive processes (assembling and moving objects, painting, welding, etc.), that justify the off-line trajectory planning. Saramago et al. [12] considered a solution to the problem of moving a robot manipulator with minimum cost along a specified geometric path. The optimal traveling time and the minimum mechanical energy of the actuators are considered together to build a multi-objective function and the results depend on the associated weighting factors. The optimization problem is subject to physical constraints. Fotouchi-C. et al. [13] used a piecewise cubic spline function to construct the joint trajectories with a given speed profile. They divided the problem into two parts: geometric trajectory planning and trajectory speed control. Dongkyoung Chwa et al. [14] presented a novel approach to an on-line trajectory planning method for the robot arms to track, intercept, and grasp moving objects under torque and velocity constraints. Piazzi A. et al. [15] developed a new approach based on interval analysis to find the global minimum-jerk trajectory of a robot manipulator within a joint space scheme using cubic splines.

In this paper, the MariBot trajectory planning is addressed. Unlike the devices designed for the rehabilitation of chronic patients, the MariBot was designed to be used with sub-acute patients. In this phase, the machine must perform passive exercises, i.e. reproduce the movements of the patient’s upper limb induced by the therapist. Therefore, the aim of this work is to optimize the trajectory planning of MariBot to obtain a trajectory of the limb at a proper distance from the patient’s body; secondly, we aim to obtain these trajectories by means of simple algorithms, in order to implement them in the real-time controller of the robot.
II. MariBot

The MariBot is a rehabilitation robot designed as an evolution of a previous prototype: the NeReBot [11]. The first robot is a 3 d.o.f. wire-based robot characterized by a manually adjustable mechanical structure which is used to support the wires (see figure 1). Three wires are used to move and sustain the splint on which the patient’s forearm is fastened, whereas a fabric stripe is used to sustain the patient’s shoulder. The therapist, after setting the configuration of the links and the linear position of the wire entry points along the links (which can be optimized according to the specific rehabilitation exercise), can record the exercise using a teaching-by-showing procedure (learning phase). After this phase, the control system produces a very smooth three-dimensional motion of the patient’s upper limb by interpolating the recorded via-points (therapy phase). NeReBot is a good compromise between robot simplicity and range of motion. In fact, since the wires originate from an overhead structure, there is a good vertical range of motion but less control of movements in the horizontal plane.

The MariBot is a 5 d.o.f. wire-based robot which maintains the NeReBot 3 d.o.f. wire-drive philosophy but exploits a 2 d.o.f. serial robotic arm to adapt wire entry points configuration to the current position of the patient’s upper limb (see figure 2). By on-line controlling the position of the serial structure, wire entry points can be moved in the horizontal plane to help the patient perform horizontal movements of the arm. In this way, not only the working space results significantly enlarged with respect to the NeReBot one, but also the pre-treatment manual set up of the machine is avoided. Like the NeReBot, the MariBot can be easily carried to be placed aside the patient (either sitting on the wheelchair or laying on the bed); however, the MariBot chassis, based on a commercial lifting device, results much lighter and less cumbersome than the NeReBot one.

The working principle of the first robot is maintained. During the learning phase, the wire gearmotors produce a constant wire tension, whereas joint motors are switched off and the overhead structure follows the patient’s arm movements induced by the therapist. Both joint rotations and wire gearmotors angular positions are recorded, defining a set of via-points. Finally, the reference trajectory for the therapy phase is obtained by interpolating the via-points in joint space. The control system is able to control the position/torque of each joint and the length/tension of each wire so both passive and active exercises can be implemented by combining these control actions.

In this work, trajectory planning of the 2 d.o.f. serial robotic arm of MariBot is addressed. Whereas wire gearmotor trajectories are obtained by simple cubic spline interpolation of the recorded via points (as in the case of NeReBot), path planning for the rotational joints of the overhead structure is more complicated. In fact, since the very simple teaching-by-showing procedure of the NeReBot is maintained (which was reported as one of the strength points of the robot during its first clinical trials), very limited information about the movement to be performed are given: each segment of the exercise is defined by the starting point and the endpoint only, both expressed in joint space (an additional speed factor can be set by the operator, which is used to scale the time base). Starting from these data, our research effort was devoted to obtain a proper spatial motion of the patient’s arm by means of very simple joint trajectories, so as to gain good machine performances with a reduced control system workload.

III. Trajectory planning in Cartesian space

First of all, a suitable Cartesian trajectory for a given point of the overhead structure was defined. For this purpose, the kinematic model of the robotic arm was developed, according to the Denavit-Hartenberg convention [16]. As shown in figure 3, three reference frames are defined, the absolute one (frame 0, to be placed just over the patient’s...
Let us consider the following expressions for the parametric curve:

- **arm opening phase** ($\rho_{fin} > \rho_{in}$):
  \[
  \rho(\alpha) = \rho_{in} + (\rho_{fin} - \rho_{in}) \left[ 1 - \cos^n \left( \frac{\pi}{2} \frac{\alpha - \alpha_{in}}{\alpha_{fin} - \alpha_{in}} \right) \right] \tag{2}
  \]

- **arm closing phase** ($\rho_{fin} < \rho_{in}$):
  \[
  \rho(\alpha) = \rho_{fin} + (\rho_{in} - \rho_{fin}) \left[ 1 - \sin^n \left( \frac{\pi}{2} \frac{\alpha - \alpha_{in}}{\alpha_{fin} - \alpha_{in}} \right) \right] \tag{3}
  \]

where $n$ is a real positive value. Figure 4 shows the influence of $n$ on the Cartesian trajectory of point $W$: high values of $n$ lead to very large trajectories, whereas for $n < 1$ wrist motion is too close to patient’s shoulder. A good compromise is given by the choice of $n = 1$.

Figure 5 shows the Cartesian trajectory of point $W$ in two typical exercises. The first one (on left side) corresponds to a rotation of the shoulder around the vertical axis ($z$ axis of absolute reference frame), the second one represents a flexion/extension of the patient’s upper limb. In both cases, $\rho_{in}$ is set to 0.3m and $\rho_{fin}$ is set to 0.5m, whereas initial and final values for angle $\alpha$ are $0^\circ$ and $120^\circ$ in the first exercise, $-10^\circ$ and $50^\circ$ in the second one. The blue curves are calculated...
lated by using equation (2) with \( n = 1 \); the red curves represent a typical joint space planned trajectory, obtained with a trapezoidal velocity profile for both joints. As is shown, the Cartesian trajectory is much better in the blue case, since shoulder rotation is performed with a suitably "close" wrist trajectory, whereas arm flexion/extension is executed on a nearly linear trajectory.

The Cartesian velocity of point \( W \) is shown in figure 6. To obtain the blue lines, a time law for \( \alpha \) must be chosen; in this case we used a trapezoidal velocity profile for \( \alpha \). Clearly, being the length of the path obtained by equation (2) smaller than the one obtained with joint space path planning, the area below the blue lines in figure 6 is smaller than the one below the red lines, i.e. we have a lower mean velocity while executing the exercise.

Figure 7 shows joint angular velocities for both exercises and with the two different path planning algorithms. Of course, the red curves are calculated directly, whereas the blue ones are obtained by inversion of the kinematics of the manipulator. In order to avoid to implement the reverse kinematics of the serial arm into the real-time controller of the machine, path planning should be performed in joint space. To this regard, we can notice that the blue velocity profile of joint #2 has a triangular shape, which indicates that the second joint is somehow "delayed" with respect to the first joint, whose velocity profile is more similar to a trapezoidal one. In fact, in order to keep the wrist of the patient close to the origin of the absolute reference frame, the motion of second joint must be delayed as much as possible (please notice that we are considering an arm opening phase, with \( \rho_{j1n} > \rho_{j2n} \)). Starting from these considerations, in the next section a time law for each joint will be defined.

IV. Trajectory planning in joint space

In this paragraph we aim to obtain a Cartesian path similar to the blue one of figure 5, but performing trajectory planning in joint space. For this purpose, a nearly trapezoidal velocity profile is used for joint #1 whereas a triangular velocity profile is used for joint #2. In this way, very simple planning equations are obtained and, thanks to the similarity of the velocity profiles to the blue ones depicted in figure 7, the trajectory of point \( W \) is very similar to the one planned in the Cartesian space.

For both joints, different time laws \( \vartheta_i(t) \) are chosen in the following time intervals:

- **acceleration phase** \( \vartheta_{i,a}(t) \):
  \[
  0 \leq t \leq T_a
  \]

- **middle phase** \( \vartheta_{i,m}(t) \):
  \[
  T_a < t < (T - T_d)
  \]

- **deceleration phase** \( \vartheta_{i,d}(t) \):
  \[
  (T - T_d) \leq t \leq T
  \]

where \( T, T_a \) and \( T_d \) are respectively total motion time, acceleration time and deceleration time (these terms would be more appropriate for a trapezoidal velocity profile law).

A. joint #1 trajectory planning

For the first joint angular position \( \vartheta_1 \), the polynomial functions are defined as follows:

\[
\begin{align*}
\vartheta_1(a)(t) &= a_1t^3 + b_1t^2 + c_1t + d_1 \\
\vartheta_1(m)(t) &= t^2 + b_2 \\
\vartheta_1(d)(t) &= a_2t^3 + b_3t^2 + c_3t + d_3
\end{align*}
\]

These laws generate a nearly trapezoidal velocity profile. In fact, the middle law has constant velocity, whereas the first and the last polynomial functions are third-order laws, so that null joint acceleration can be imposed both at the beginning and at the end of motion. To calculate the polynomial coefficients, the following constraints are given:

\[
\begin{align*}
\vartheta_1(a)(0) &= \vartheta_{1,m}(0) \\
\vartheta_1(a)(0) &= 0 \\
\vartheta_1(a)(0) &= 0 \\
\vartheta_1(a)(T_a) &= \vartheta_{1,m}(T_a) \\
\vartheta_1(a)(T_a) &= \vartheta_{1,m}(T_a) \\
\vartheta_1(m)(T - T_d) &= \vartheta_{1,d}(T - T_d) \\
\vartheta_1(m)(T - T_d) &= \vartheta_{1,d}(T - T_d) \\
\vartheta_1(d)(T) &= \vartheta_{1,f1n} \\
\vartheta_1(d)(T) &= 0 \\
\vartheta_1(d)(T) &= 0
\end{align*}
\]
By substituting the expressions (4) into system (5) and solving for the polynomial coefficients, the following expressions for joint #1 angular position, velocity and acceleration are obtained:

\[
\begin{align*}
\dot{\vartheta}_{1,a}(t) &= \dot{\vartheta}_{1,\text{in}} + \frac{v_{\text{max}}}{3T_a} t^3 \\
\dot{\vartheta}_{1,m}(t) &= \dot{\vartheta}_{1,\text{in}} + v_{\text{max}} (t - \frac{2T_a}{T_d}) \\
\dot{\vartheta}_{1,d}(t) &= \dot{\vartheta}_{1,\text{fin}} + \frac{v_{\text{max}}}{3T_d} (T - T)^3 \\
\end{align*}
\]

(6)

\[
\begin{align*}
\ddot{\vartheta}_{1,a}(t) &= \frac{V_{\text{max}} t^2}{T_a} \\
\ddot{\vartheta}_{1,m}(t) &= V_{\text{max}} \\
\ddot{\vartheta}_{1,d}(t) &= \frac{2V_{\text{max}}}{T_d} (t - T) \\
\end{align*}
\]

(7)

\[
\begin{align*}
\dddot{\vartheta}_{1,a}(t) &= \frac{2V_{\text{max}}}{T_a} \\
\dddot{\vartheta}_{1,m}(t) &= 0 \\
\dddot{\vartheta}_{1,d}(t) &= \frac{2V_{\text{max}}}{T_d} \\
\end{align*}
\]

(8)

where

\[
v_{\text{max}} = \frac{\dot{\vartheta}_{1,\text{fin}} - \dot{\vartheta}_{1,\text{in}}}{T - \frac{2}{3}(T_a + T_d)}
\]

(9)

B. Joint #2 Trajectory Planning

The angular position of second joint \(\vartheta_2(t)\) is defined as follows:

\[
\begin{align*}
\vartheta_{2,a}(t) &= a_1 t^3 + b_1 t^2 + c_1 t + d_1 \\
\vartheta_{2,m}(t) &= a_2 t^2 + b_2 t + c_2 \\
\vartheta_{2,d}(t) &= a_3 t^3 + b_3 t^2 + c_3 t + d_3 \\
\end{align*}
\]

(10)

Unlike \(\vartheta_{1,m}(t)\), the expression of \(\vartheta_{2,m}(t)\) is chosen as a second-order polynomial to obtain a linear velocity profile. So, a new constraint must be added to system (5), in which subscript 1 is to be substituted with 2. Moreover, different time laws (and additional constraints) must be used in the case of arm opening and arm closing trajectories:

- **Arm opening phase**, additional condition \(\dot{\vartheta}_{2,m}(0) = 0\)

\[
\begin{align*}
\dot{\vartheta}_{2,a}(t) &= \dot{\vartheta}_{2,\text{in}} + \frac{V_{\text{max}}}{3T_a(T_a + T_d)} t^3 \\
\dot{\vartheta}_{2,m}(t) &= \dot{\vartheta}_{2,\text{in}} + \frac{V_{\text{max}}(3T_a - T_d)}{6} + \frac{V_{\text{max}}}{2(T_a - T_d)} t^2 \\
\dot{\vartheta}_{2,d}(t) &= \dot{\vartheta}_{2,\text{fin}} + \frac{V_{\text{max}}}{3T_d} (T - T)^3 \\
\end{align*}
\]

(11)

\[
\begin{align*}
\ddot{\vartheta}_{2,a}(t) &= \frac{V_{\text{max}}}{T_a(T_a + T_d)} t^2 \\
\ddot{\vartheta}_{2,m}(t) &= \frac{V_{\text{max}}}{T_a(T_a + T_d)} t \\
\ddot{\vartheta}_{2,d}(t) &= \frac{V_{\text{max}}}{T_d} (t - T)^2 \\
\end{align*}
\]

(12)

\[
\begin{align*}
\dddot{\vartheta}_{2,a}(t) &= \frac{2V_{\text{max}}}{T_a(T_a + T_d)} t \\
\dddot{\vartheta}_{2,m}(t) &= \frac{2V_{\text{max}}}{T_a(T_a + T_d)} \\
\dddot{\vartheta}_{2,d}(t) &= \frac{2V_{\text{max}}}{T_d} (t - T) \\
\end{align*}
\]

(13)

where

\[
v_{\text{max}} = \frac{6(\dot{\vartheta}_{2,\text{fin}} - \dot{\vartheta}_{2,\text{in}})(T - T_a)}{3T_a^2 - 4T_a T_d + T_d^2}
\]

(14)

- **Arm closing phase**, additional condition \(\dot{\vartheta}_{2,m}(T) = 0\)

\[
\begin{align*}
\dot{\vartheta}_{2,a}(t) &= \dot{\vartheta}_{2,\text{in}} + \frac{V_{\text{max}}}{3T_a} T_a^2 + \frac{V_{\text{max}} T_a (4T - T_a)}{6(T - T_a)} + \frac{V_{\text{max}} t}{T - T_a} \\
\dot{\vartheta}_{2,m}(t) &= \dot{\vartheta}_{2,\text{fin}} + \frac{V_{\text{max}}}{3T_d(T - T_a)} (T - T)^3 \\
\dot{\vartheta}_{2,d}(t) &= \dot{\vartheta}_{2,\text{in}} + \frac{V_{\text{max}}}{T_d(T - T_a)} (t - T) \\
\end{align*}
\]

(15)

\[
\begin{align*}
\ddot{\vartheta}_{2,a}(t) &= \frac{V_{\text{max}}}{T_a} t^2 \\
\ddot{\vartheta}_{2,m}(t) &= \frac{V_{\text{max}} T_a}{T - T_a} - \frac{V_{\text{max}} t}{T - T_a} \\
\ddot{\vartheta}_{2,d}(t) &= \frac{V_{\text{max}}}{T_d(T - T_a)} (t - T) \\
\end{align*}
\]

(16)

\[
\begin{align*}
\dddot{\vartheta}_{2,a}(t) &= \frac{2V_{\text{max}}}{T_a} t \\
\dddot{\vartheta}_{2,m}(t) &= \frac{2V_{\text{max}}}{T_d(T - T_a)} \\
\dddot{\vartheta}_{2,d}(t) &= \frac{2V_{\text{max}}}{T_d} (t - T) \\
\end{align*}
\]

(17)

where

\[
v_{\text{max}} = \frac{6(\dot{\vartheta}_{2,\text{fin}} - \dot{\vartheta}_{2,\text{in}})(T - T_a)}{3T_a^2 - 4T_a T_d + T_d^2}
\]

(18)
Figure 8 and 9 allow to compare, respectively for the arm opening phase and the arm closing phase, the joint angular velocities calculated with the Cartesian path planning (blue lines) to the ones obtained with the joint space planning (green lines). First of all, we can notice that the green lines present null slope both at the beginning and at the end of motion (this condition was not imposed for the time law of parameter \( \alpha \) in the Cartesian planning). Secondly, the triangular velocity profile for joint #2 is right-left flipped in the arm closing phase; in fact, in this case the motion of second joint must be anticipated with respect to joint #1, to obtain a close motion of the patient’s wrist.

Figure 10 shows the Cartesian trajectory of point W during the arm opening phase for both shoulder rotation and arm flexion-extension. As it shows, the green trajectories are very similar to the ones obtained with the Cartesian path planning (they are even closer to the origin than the blue lines). Moreover, the velocity profile of point W, depicted in figure 11, is better in the green case, since it provides less speed variations in the middle time interval. As a result, a more comfortable motion for the patient is obtained.

Figure 10. Cartesian trajectory of point W during shoulder rotation (left side) and arm flexion-extension (right side). The blue lines are the same of figure 5, the green lines are calculated using the joint space path planning.

Figure 11. Cartesian velocity of point W during shoulder rotation (left side) and arm flexion-extension (right side). The green lines are more similar to a trapezoidal profile, producing a more comfortable motion for the patient.

V. Conclusions

The trajectory planning of a two degrees of freedom serial robotic arm was presented. Due to the particular application field of the robot, i.e. the upper limb rehabilitation of post-stroke patients, major attention was devoted to obtain comfortable trajectories for the patient’s arm. The trajectories have been defined in the Cartesian space firstly, in order to meet the specifications on patient’s arm motion.

Secondly, a joint space interpolated trajectory has been calculated with a comparable Cartesian motion, obtaining very simple planning laws to be implemented in the real-time controller of the machine. Dynamic analysis was not presented here, since motion speed is very low and no significant results can be obtained in terms of joint torques by changing joint trajectories. Moreover, the joint torques required by the voluntary motion of the patient are an order of magnitude greater than the ones which can be calculated on the planned trajectories.

The MariBot is currently undergoing lab tests at the Robotics Laboratory of DIMEG.

References


