Dynamic Model of a 6-DOF Parallel Robot by Considering Friction Effects

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Abstract—The dynamic analysis is the basic element of the mechanical design and control of parallel mechanisms. This paper presents a method for the dynamical modeling of the guided in three points parallel robots with six degrees of freedom having triangular platform, which include the friction. The obtained dynamical algorithms, based on the kinematical algorithms, offer the possibility of a more complex dynamic study for these parallel robots in comparison with the existing simple dynamic models without friction.

Keywords: parallel robots, kinematics, dynamics, friction

1. Introduction

Parallel robots have some advantages over serial ones such as high stiffness, very good precision, high speeds and accelerations. However, kinematics and dynamics analysis of the parallel structures is much more complicated due to the presence of the constraints and singularities. Dynamics effects and analysis are the basis of design specification and advanced control of parallel mechanical systems. To establish the equations of motion, essentially there are four methods: Newton-Euler equations; Lagrange equations of first kind with so-called Lagrange multipliers; Lagrange equations of second kind with a minimum number of system coordinates; Virtual work formulation.

In [1], a simplified method of determining the dynamic model of the HEXA robot in two steps is proposed. It is considered that there is no friction in spherical joints and that the parallelogram lower bars mass is concentrated at the extremities and they do not have inertia moments. Codourey [2] proposes the first dynamic model that can be used to control the parallel DELTA robot in real time. Guglielmetti and Longchamp [3] present the inverse dynamic model for the DELTA robot in the analytical form using the Newton’s laws. Tadokoro [4] also explains how dynamic equations can be used in a control scheme closed loop for serial robots. Stamper and Tsai [5] present a dynamical model for a parallel structure with three degrees of freedom. This model was also generated with the simplications of the Codourey model. Honneger et al. [6] suggested the use of the dynamic equations in an adaptive control scheme for the Hexaglide robot, in which the pursuance errors are used on-line to correct the parameters used in dynamic equations. Guégan [7] presents a new solution for the dynamic model for the Orthoglide with Newton-Euler equations. Geng and Haynes [8] and Liu [9] use the Lagrange formalism to derive the dynamical equations of the parallel robots. Miller and Clavel [10] present the complete dynamic model of the DELTA robot based on Lagrange equations. In this case one considers that the robot bars possess inertia moments themselves. This allows the estimation of the necessary driving moments for a given trajectory of the platform. To solve the dynamic model, Merlet [11] uses Lagrange formulas. He has applied the direct and the inverse dynamic model for the “left hand”, a prototype accomplished at INRIA based on a KPS kinematic chain structure. Pisla and Kerle [12] propose a generalized dynamic model for parallel robots using first order Lagrange equations on the basis of equivalent masses. Itul and Pisla [13] present a comparative study among various dynamical methods for parallel robots regarding certain parameters. In [14], different solutions for solving the dynamical model for the guided in three points parallel robots are presented. Kovèceses et al. [15] intend to address dynamic models from an unified analytical point of view, which is based on differential variational principles.

Generally, in the above cited contributions, the dynamic equations are obtained without considering all the actions on a parallel manipulator. Until now, the experimental identification of dynamics for the parallel robots is restricted to simple models in combination with adaptive control algorithms. The most important force source which is not included is the friction [16]. Few authors have also approached the dynamic model considering the friction forces too. This gap was closed by a new approach presented in [17], where Grotjahn develops a complex dynamic model including friction, which uses Jourdain’s principle of virtual power and is suited for the parallel robot control of the innovative hexapod PaLiDA. The friction model includes friction in active joints as well in passive joints. Poignet et al. [18] deal with the application of the interval analysis to estimate the 4-degrees of freedom parallel robot dynamic parameters. Yiu et al [19] developed the dynamic modeling including friction for a planar 2-dof redundant
parallel robot. Riebe et al. [20] present an dynamic model for a Stewart platform with six DOF, based on Newton-Euler equations including the frictional behaviour. The parameters describing the friction model are identified and optimized. Chen et al. [21] present a dynamic model of a Cartesian guided tripod including the nonlinear compliance and mechanical friction. Quantitative analysis and comparison of the variation friction sources from the actuated joints and passive joints have been conducted.

It is estimated that the forces due to the friction represent about 25% from the forces/torques which are necessary for the manipulator moving in the typical situations [16]. For the friction forces approximation, two models could be applied:

- the model of the viscous friction in which the force/torque due to the friction is in proportion to the joint movement speed:

\[ Q_{f} = c \dot{q} \tag{1} \]

where \( c \) is the viscous friction coefficient (constant).

- the model of Coulomb friction in which the force/torque due to the friction is in proportion to the normal reaction on joint axis:

\[ Q_{f} = \mu N \frac{d}{2} \text{sgn}(\dot{q}) \tag{2} \]

for the rotational joint

\[ Q_{f} = \mu N \text{sgn}(\dot{q}) \tag{3} \]

for the translational joint

where

\( \mu \) is the bearing friction coefficient;
\( d \) is the spindle diameter from the rotation joint;
\( \mu \) is the dry friction coefficient.

These two models together lead to a more reasonable approximation of the effects due to the friction:

\[ Q_f = Q_{f_v} + Q_{f_c} \tag{4} \]

The parallel mechanisms also contain the passive joints. The forces/torques to be applied the kinematical axes must be supplemented with the friction effect from these passive joints.

In this paper a method for the supplementary forces evaluation from the drive joints due to the friction from the drive and passive joints is proposed. Similar to [17], [20], [21], the present method considers the whole mechanism, the generalized additional forces which include the friction effect from the corresponding drive joints and the friction effect from all passive joints. The passive joints are approximated with compliant joints whose momentum is equal to the friction torques of these joints.

In comparison with dynamic models from [17], [20], the evaluation of the generalized forces is performed in two stages. In the first stage, the drive generalized forces are established, based on the dynamical equations without friction. Using the Newton-Euler equations, the reactions from each joint are computed. In the second stage the additional drive generalized forces due to the frictions from the active and passive joints are computed.

The paper is organized as follows: Section II is dedicated to the kinematic modeling of the parallel robot with six degrees of freedom; Section III deals with dynamical modeling with computation of the additional drive generalized forces due to the frictions; Section IV presents some numerical results on a parallel robot. The conclusions of this work are found in the section V.

II. Kinematic modeling of guided in three points parallel structures

Plitea [22] defines the spatial parallel symmetrical and guided mechanisms. The structural scheme of the A type guided in three points parallel mechanisms with 6 DOF is presented in Figure 1. The circles surrounding the joints, which are located in the proximity of the fixed base suggest that these joints are actuated, each of them having 1 DOF. Each kinematic chain, which connects the fixed base with the mobile platform, also contains a passive joint with 1 DOF and a passive spherical joint with 3 DOF. The spherical joints centers are conventionally called guiding points. Each of them is moving with respect to the fixed base on a mobile 2-DOF curve.

The general scheme of an “i” kinematic chain \((i=1, 2, 3)\) is illustrated in Figure 2. The actuating joints with 1 DOF could be “R”-revolute or “P”-prismatic. To avoid the mechanism locking, the passive joints situated on level III are of rotational type. The variables and constant parameters of a kinematic chain are represented by the Denavit-Hartenberg notations. Each kinematic chain has 15 parameters: 12 constant parameters, two controlled variable parameters \((q_i = \theta_i \text{ or } q_i = d_i)\) and \(q_{i+3} = \theta_{i+3} \text{ or } q_{i+3} = d_{i+3}\) and one passive variable parameter \((u_i = \theta_{i+6})\).

Fig. 1. The structural archetype scheme
Thus, four structural schemes could be derived: 3-RRRS, 3-RPRS, 3-PRRS, 3-PPRS. As an application, it was chosen the PPRS parallel manipulator (figure 3). The points $B_i$ (i=1,2,3) and $E_i$ (i=1,2,3) represent active prismatic joints, whereas the revolute joints and ball joints are located in $C_i$ (i=1,2,3) respectively in $A_i$ (i=1,2,3).

The six prismatic joints are actuated, the rest of them are passive. The geometric parameters $R$, $r$ and $l$ represent: the radius of fixed base, radius of the working platform (WP) and length of the guiding rods $C_i A_i$. The angles $\delta_1^i = \delta_1 = 0^\circ$, $\delta_2^i = \delta_2 = 120^\circ$, $\delta_3^i = \delta_3 = 240^\circ$ represent the angles between kinematic chains from the WP and the fixed base level. In Figure 4, the CAD model is presented.

To solve the geometric problems, Iul and Pisla [23] use the method of input-output (I-O) equations. The I-O equations are the relations between the joint coordinates ($q_i$ and $q_{i+3}$; i=1,2,3) and the world coordinates of the working platform (WP), $X,Y,Z,\alpha,\beta,\gamma$:

$$
\begin{align*}
(\overrightarrow{P_l} - \overrightarrow{P_{C_l}})^2 = l^2 &= 0; \\
(\overrightarrow{P_l} - \overrightarrow{P_{C_l}}) \cdot \overrightarrow{n_i} &= 0; \quad i = 1,2,3
\end{align*}
$$

where: $\overrightarrow{P_l}(X_l,Y_l,Z_l)$ is radius vector from the base frame center $O$ to the guiding point $A_i$; $\overrightarrow{P_{C_l}}(X_{C_l},Y_{C_l},Z_{C_l})$ is radius vector from the point $O$ to joint $C_i$ center; $\overrightarrow{n_i}$ is unit vector of $C_i$ joint axis.

In this case the inverse geometrical model has an analytical solution, which involves a second degree algebraic equation solution and the geometrical direct model can be only numerically solved by using the Newton-Raphson method.

To find the kinematic model (KM), the I-O equations are derived with respect to time yielding to:

$$
[A]\ddot{q} = [B]X
$$

where: $\ddot{q}(\dot{q}_1,\dot{q}_2,\dot{q}_3,\dot{q}_4,\dot{q}_5,\dot{q}_6)$ is the driving velocity vector, $\ddot{X}(X,\dot{Y},\dot{Z},\omega_X,\omega_Y,\omega_Z)$ is the end-effector velocity vector, $[A]$ is the inverse Jacobi matrix, $[B]$ is the direct Jacobi matrix.

$$
[A] = 
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & Z_{C_1} - Z_{C_l} & 0 & 0 \\
0 & 0 & 0 & 0 & Z_{C_2} - Z_{C_l} & 0 \\
0 & 0 & 0 & 0 & 0 & Z_3 - Z_{C_3}
\end{bmatrix}
$$
\[
[B] = \begin{bmatrix}
N_1^r \\
N_2^r \\
N_3^r \\
(p_1 - p_1^c_1)^r \\
(p_2 - p_2^c_1)^r \\
(p_3 - p_3^c_1)^r \\
\end{bmatrix} \times \begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
(p_1 - p_1^c_1) \\
(p_2 - p_2^c_1) \\
(p_3 - p_3^c_1) \\
\end{bmatrix}
\]

\(\bar{P}(X,Y,Z)\) is the radius vector from the point \(O\) to the \(o\) mobile platform center. While the inverse kinematic model \((\ddot{q} = [A]^{-1} [B][\dot{X}] )\) has an analytical solution, the direct kinematic model \(( \dot{X} = [B]^{-1} [A][\ddot{q}] )\) can be only numerically solved, i.e. it requires the discrete definition of the matrix \([B]\) for the mobile platform pose parameters and then its numerical inversion.

III. Dynamics with computation of the additional drive generalized forces due to the friction

A friction torque in a passive joint is a vector, situated along the relative rotational speed of the considered link with respect to the neighboring links and which has opposite direction to this relative speed (figure 5):

\[
\begin{align*}
M_n &= -\left( c_\nu \omega_n + \mu_s \frac{d_s}{2} \sqrt{H_i^2 + V_i^2 + W_i^2} \right) = -k_n \tilde{\omega}_n \\
M_{ri} &= -\left( c_\nu \dot{u}_i + \mu_r \frac{d_r}{2} \sqrt{(V_i^*)^2 + (W_i^*)^2} \right) = -k_r \dot{u}_i
\end{align*}
\]

The following notations were made:

- \(H_i, V_i, W_i\) are the reaction force components from the spherical joint considering the model without friction;
- \(\bar{M}_{ni}\) is the friction moment from the spherical joint \(A_i\);
- \(c_\nu\) is the viscous friction coefficient from the spherical joint;
- \(\mu_s\) is the Coulomb friction coefficient from the spherical joint;
- \(d_s\) is the diameter of the spherical joint ball;
- \(\tilde{\omega}_n = \tilde{u}_i - \tilde{\omega}\) is the relative angular speed of the guiding bar \(A_iC_i\) with respect to the mobile plate;
- \(\tilde{u}_i = u_i \bar{n}_i\) is the absolute angular speed of the guiding bar \(A_iC_i\);
- \(\tilde{\omega}\) is the absolute angular speed of the mobile plate;
- \(\bar{M}_{ri}\) is the friction moment from the passive rotation joint \(C_i\);
- \(c_r\) is the viscous friction coefficient from the rotational joint \(C_i\);
- \(\mu_r\) is the Coulomb friction coefficient from the joint \(C_i\);
- \(d_r\) is the diameter of the rotational joint spindle;

\[
\begin{align*}
H_i^* &= H_i + m_b \ddot{q}_i \\
V_i^* &= V_i \tilde{u}_i - W_i \ddot{u}_i + \frac{1}{2} m_b (\ddot{u}_i \tilde{c}_u - \ddot{u}_i^2 \tilde{u}_i) + m_b (g + \ddot{q}_i) \\
W_i^* &= -V_i \tilde{c}_u - W_i \ddot{u}_i + \frac{1}{2} m_b l (\tilde{u}_i \tilde{u}_i \ddot{u}_i + \ddot{u}_i^2 \tilde{c}_u)
\end{align*}
\]

where

- \(m_b\) is the guiding bar mass;
- \(g\) is gravity acceleration;
- \(c\) is cosine; \(s\) is sinus.

The approached method assumes the propagation of the virtual displacements applied to the drive joints to all passive joints of the mechanism, respectively that a displacement of an active joint involves displacements in all the passive joints of the mechanism.

Through the differentiation of the closure equations for the direct geometrical model one obtains the relation between the drive joint speeds and the passive joint speeds:

\[
[A]_{n} \ddot{q} = [B]_{n} \ddot{u}; \quad [J_{n}] \ddot{q}; \quad [J_{n}] = [B]_{n}^{-1} [A]_{n}
\]
\[ \tilde{q} = [\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_6] \, , \quad \ddot{u} = [\ddot{u}_1, \ddot{u}_2, \ddot{u}_3] \]

The direct kinematical model has the form:

\[ \bar{X}_i = [J_i] \tilde{q} - [J_{q_i}] [B]^{-1} \cdot [A] \]

where:

\[ \bar{X}_i = [\bar{X}, \bar{Y}, \bar{Z}, \omega_X, \omega_Y, \omega_Z] \]

In order to get the generalized drive forces which are necessary to overcome the frictions from the passive joints the principle of the virtual power is applied:

\[ \Delta \tilde{Q}^p = \tilde{u} + \sum_{i=1}^{3} \Delta \tilde{M}_{ri} \cdot \ddot{u}_i + \sum_{i=1}^{3} \Delta \tilde{M}_{qi} \cdot \ddot{\omega}_r = 0 \]  

Finally, it yields:

\[ [\Delta Q_1^y, \Delta Q_2^y, \Delta Q_3^y, \Delta Q_1^z, \Delta Q_2^z, \Delta Q_3^z] = [J_{q_i}] \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \]

The necessary drive generalized forces for the specified displacement performance have the form:

\[ Q_i^m = \bar{Q}_i + \Delta Q_i^m + \Delta Q_i^p, \quad i = 1, 2, 3 \]  

\[ Q_{ri}^m = \bar{Q}_{ri} + \Delta Q_{ri}^m + \Delta Q_{ri}^p, \quad i = 1, 2, 3 \]

where \( Q_i, Q_{ri} \) are the generalized drive forces, which have been obtained with the inverse dynamical model without friction and \( \Delta Q_i^m \) and \( \Delta Q_{ri}^m \) are the additional forces due to the frictions from the active joints:

\[ \Delta Q_i^m = c_i \dot{q}_i + \mu \sqrt{[V_i]^2 + [W_i]^2 + [(m_i + m_{ri})g]^2} \cdot \text{sgn}(\dot{q}_i) \]  

\[ \Delta Q_{ri}^m = c_i \dot{q}_{ri} + \mu \sqrt{[V_{ri}]^2 + [W_{ri}]^2} \cdot \text{sgn}(\dot{q}_{ri}) \]

where \( c_i \) is the viscous friction vector from the active prismatic joint; \( \mu \) is the sliding friction coefficient from the same joint.

If the speed of the drive joint is null, for \( \text{sgn}(\dot{q}_i) \), the joint motion tendency will be considered, i.e. the sign of the next subsequent speed.

### IV. Some simulation results

The new dynamic algorithms including friction effects have been implemented in a simulation program for parallel robots. For the simulation, the parallel mechanism with the fixed base at the bottom was considered.

The chosen input data for the parallel robot are:

- \( R = 0.15 \text{ m} \), \( l = 0.30 \text{ m} \), \( r = 0.10 \text{ m} \), \( \delta_1 = \delta_1' = 0^\circ \), \( \delta_2 = \delta_2' = 120^\circ \), \( \delta_3 = \delta_3' = 240^\circ \), \( M = 1 \text{ kg} \), the manipulated object mass \( m = 1 \text{ kg} \), the actuators masses:
- \( m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = 0.5 \text{ kg} \), the guiding bars masses \( m_b = 0.2 \text{ kg} \).

The selected displacement of the working platform is a helical translation:

\[ \lambda(t) = 0.5 + \frac{\lambda(t)}{2 \cdot \pi} \]

where

\[ \lambda(t) = \begin{cases} \frac{2}{\pi} \cdot \left( t - \frac{\pi}{2} \right) + \frac{\pi}{2} & \text{if } 0 \leq t < \frac{\pi}{2} \\ 2 \cdot \left( t - \frac{\pi}{2} \right) + 2 \cdot (t - \pi) + 3 \cdot \frac{\pi}{2} & \text{if } \frac{\pi}{2} \leq t \leq \frac{3\pi}{2} \\ 2 \cdot (t - \pi)^2 + 2 \cdot (t - \pi) + 3 \cdot \frac{\pi}{2} & \text{if } \frac{3\pi}{2} \leq t \leq 2 \pi \end{cases} \]

\[ \psi = \theta = \varphi = 0 \]

In Figure 6, there are represented the generalized drive forces \( Q_{1n} \) and \( Q_{1n}^m \) for the motor 1 considering the simplified model without friction, respectively the new model considering the friction forces.

The following input data have been considered:

- \( c_i = 0.0015 \text{ N} \cdot \text{s} / \text{m} \); \( c_r = 0.02 \text{ N} \cdot \text{m} / \text{s} / \text{rad} \);
- \( c_s = 0.02 \text{ N} \cdot \text{m} / \text{rad} / \text{rad} \); \( \mu = 0.05 \); \( d_r = 0.01 \text{ m} \);
- \( d_s = 0.016 \text{ m} \); \( \mu_p = 0.1 \); \( s = 0.00005 \text{ m} \);

\[ \mu_p = \frac{\mu_p + \frac{2s}{d_r}}{1 + \mu_p}; \quad \mu_s = \frac{1}{3} \mu_p \]

![Fig. 6. The generalized drive forces \( Q_{1n} \) and \( Q_{1n}^m \)](image-url)
In Figure 7, the additional forces due to the friction in the motor 1 are represented.

![Figure 7. The additional forces](image)

The obtained numerical results have shown that the frictions from the drive joints have the substantial effect in the additional drive forces components. The friction coefficients from the passive joints do not significantly influence the additional drive forces.

V. Conclusion

The dynamic modeling of 6 DOF parallel robots considering the friction effects is a difficult problem. The method approached in the paper considers the whole mechanism, the generalized additional forces which include the friction effect from the corresponding drive joints and the friction effect from all passive joints of the mechanism. The dynamical algorithms including friction and based on kinematical algorithms offer the possibility of a more complex dynamic study of these parallel robots in order to evaluate their dynamic capabilities and to generate the control algorithms.

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References


